Development of a Simple Numerical Model to Study the Heat Transfer Performance of Li-Ion Battery

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Abstract—**Overheating when charging and discharging is a practical problem for the battery nowadays, and using phase change material (PCM) is one of the ways of cooling the batteries. However, there are many types of PCM, and the thermal properties of PCM are different from one another. In this research project, a sensitivity analysis was conducted using the thermo physical properties of battery and PCM to identify the sensitive parameters that are responsible for the increase of battery temperature. The procedure of the heat transfer in the battery and PCM is simulated using MATLAB to show how the temperature changes when the parameters of the battery and PCM are changed. The conductivity of the battery and PCM, the heat generation rate, the height and radius of the battery are sensitive parameters for the temperature in the battery. The melting of PCM has little effects on the temperature as PCM acts as a heat sink for the normal battery. The material with high thermal conductivity, high density and high heat capacity can be used for cooling the battery, and the battery with high thermal conductivity, small heat generation, small radius and height can be chosen to prevent the temperature from raising sharply. When the heat generation is extremely big, PCM with high thermal conductivity, large latent heat and suitable melting temperature can be used to cool down the battery.**

Keywords-component; heat transfer; Li-ion battery; key parameters; simluation

I. INTRODUCTION

The lithium-ion (Li-ion) battery is a type of rechargeable battery. The higher power density and efficiency, lower self-discharge rate, longer life are the advantages of this new technology and it attracts the attention of many researchers and manufacturers [1]. The chemical reactions in the Li-ion battery are

Positive reaction $2e^- + Li^+ + CoO_2 \leftrightharpoons LiCoO_2$

Negative reaction $LiC_6 \leftrightharpoons Li^+ + C_6 + e^-$

Total reaction $CoO_2 + LiC_6 \leftrightharpoons LiCoO_2 + C_6$ [1]

for the battery using $LiCoO_2$ and

$Li_xC_6 \leftrightharpoons C_6 + xLi^+ + xe^-$ at anode/electrolyte interface

$Li_{y-x}Mn_2O_4 + xLi^+ + xe^- \leftrightharpoons Li_yMn_2O_4$ at the cathode/electrolyte interface [2]

for the battery using Li_yMnO_4.

However, overheating of the Li-ion battery becomes one of the biggest problems in the recent years. The heat will be generated from the electrochemical reactions during charge and discharge of the battery as well as Joule heating as there is internal resistance for the battery. The heat has to be dissipated properly, otherwise the battery will be overheated, which may cause thermal runaway in the worst-case scenario [3].Thermal management of Li-ion batteries plays a very important role in the application of the battery to keep the battery safe and improving the performance of the as the performance of the battery is affected by the operating temperature to a very large extent [4]. Thus, it is important to study the heat transfer performance of the Li-ion battery and protect the Li-ion battery from overheating. Using PCM is one of the potential options.

Using PCM is a preferable way to cool down the battery than the air cooling [5]. Thus, the Li-ion battery can be maintained at an optimum operating temperature with suitable thermal management by using PCM in the battery. A large amount of heat can be absorbed by PCM because of its high latent heat of fusion. When the temperature of the battery exceeds the melting point of the PCM, it starts to melt and the heat generated by the battery is absorbed by PCM. Due to high latent heat of, PCM will absorb a lot of heat and protect the battery from raising too high temperature and overheating [4].

II. NUMERICAL MODEL

The battery is divided into n segments and the battery and PCM is divided into p segments in total like figure 1. It is well known that the rate of heat transfer by conduction is $Q_{conduction}=kA\frac{\Delta T}{d}$, and the rate of heat transfer by convection is $Q_{convection}=h_0 A\Delta T$. After substitute each part into the equation $q_{in}-q_{out} + E = q_{absorb}$, the following equations for rate the temperature change of each segment can be obtained. Before melting of PCM:

For the battery:

For the first control element (i=1):

$$\frac{dT_1}{dt} = -\left(\frac{2k_1}{\rho_1 c_1 \Delta r^2} + \frac{h_{up} + h_{down}}{\rho_1 c_1 l}\right)T_1 + \frac{2k_1}{\rho_1 c_1 \Delta r^2}T_2 + \frac{el + h_{up}T_{amb} + h_{down}T_{amb}}{\rho_1 c_1 l}$$

For the general control element (i=2 to n-1):

$$\frac{dT_i}{dt} = \frac{2k_1(i-1)}{\rho_1 c_1 \Delta r^2(2i-1)}T_{i-1} - \left(\frac{2k_1}{\rho_1 c_1 \Delta r^2} + \frac{h_{up} + h_{down}}{\rho_1 c_1 l}\right)T_i + \frac{2k_1 i}{\rho_1 c_1 \Delta r^2(2i-1)}T_{i+1} + \frac{el + h_{up}T_{amb} + h_{down}T_{amb}}{\rho_1 c_1 l}$$

For the last control element (i=n):

$$\frac{dT_n}{dt} = \frac{2k_1(n-1)}{\rho_1 c_1 \Delta r^2 (2n-1)} T_{n-1} - \left(\frac{2k_1(n-1) + 2k_2 n}{\rho_1 c_1 \Delta r^2 (2n-1)} + \frac{h_{up} + h_{down}}{\rho_1 c_1 l}\right) T_n + \frac{2k_2 n}{\rho_1 c_1 \Delta r^2 (2n-1)} T_{n+1} + \frac{el + h_{up} T_{amb} + h_{down} T_{amb}}{\rho_1 c_1 l}$$

For the PCM

For the first control element (i=n+1):

$$\frac{dT_{n+1}}{dt} = \frac{2k_1 n}{\rho_2 c_2 \Delta r^2 (2n+1)} T_n - \left(\frac{2k_1 n + 2k_2(n+1)}{\rho_2 c_2 \Delta r^2 (2n+1)} + \frac{h_{up} + h_{down}}{\rho_2 c_2 l}\right) T_{n+1} + \frac{2k_2(n+1)}{\rho_2 c_2 \Delta r^2 (2n+1)} T_{n+2} + \frac{h_{up} T_{amb} + h_{down} T_{amb}}{\rho_2 c_2 l}$$

For the general control element (i=n+2 to p-1):

$$\frac{dT_i}{dt} = \frac{2k_2(i-1)}{\rho_2 c_2 \Delta r^2 (2i-1)} T_{i-1} - \left(\frac{2k_2}{\rho_2 c_2 \Delta r^2} + \frac{h_{up} + h_{down}}{\rho_2 c_2 l}\right) T_i + \frac{2k_2 i}{\rho_2 c_2 \Delta r^2 (2i-1)} T_{i+1} + \frac{h_{up} T_{amb} + h_{down} T_{amb}}{\rho_2 c_2 l}$$

For the last control element (i=p):

$$\frac{dT_p}{dt} = \frac{2k_2(p-1)}{\rho_2 c_2 \Delta r^2 (2p-1)} T_{i-1} - \left(\frac{2k_2(p-1) + 2hp\Delta r}{\rho_2 c_2 \Delta r^2 (2p-1)} + \frac{h_{up} + h_{down}}{\rho_2 c_2 l}\right) T_i + \frac{2hpT_{amb}}{\rho_2 c_2 \Delta r (2p-1)} + \frac{h_{up} T_{amb} + h_{down} T_{amb}}{\rho_2 c_2 l}$$

When PCM starts melting, the corresponding equations are modified accordingly. For the liquid part of PCM, c_2 is changed to c_3 and k_2 is changed to k_3. When PCM starts melting, the corresponding equations are modified accordingly. For the liquid part of PCM, c_2 is changed to c_3 and k_2 is changed to k_3.

When the i^{th} segment of PCM is melting as shown in figure 2:

The condition of melting: $T_i \geq T_{melt}$, $Q_i < q\rho_2 \frac{(2i-1)}{2} \theta l \Delta r^2$

$\frac{dT}{dt} = 0$

$E_{absorb} = k_3(i-1)\theta l (T_{i-1} - T_i) - k_2 i \theta l (T_i - T_{i+1})$

The condition of the i^{th} segment of PCM finish melting:

$$Q_i \geq q\rho_2 \frac{(2i-1)}{2} \theta l \Delta r^2$$

The equations change accordingly.

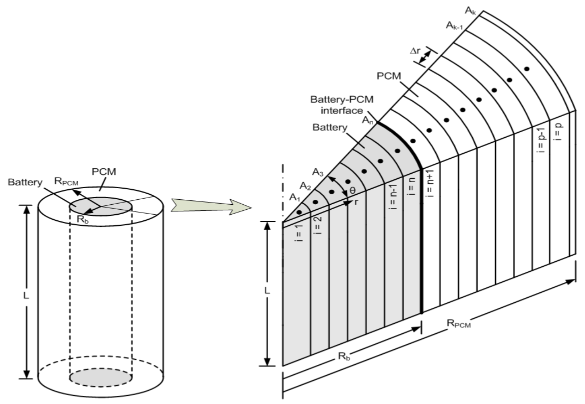

Figure 1: The battery and PCM

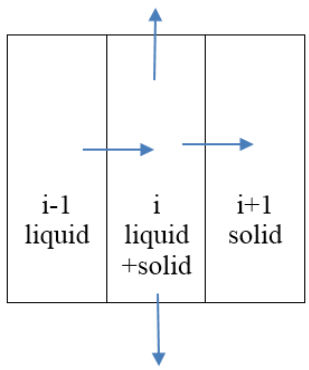

Figure 2: PCM is melting

Following assumptions are made to simplify the numerical model:

1: There is no heat transfer by radiation to the ambient.
2: The ambient temperature is constant.
3: The temperature of each control volume is uniform, i.e. there is no heat conduction in the vertical direction.
4: The conductivity of PCM when PCM is melting is $\frac{k_2+k_3}{2}$.
5: All the parameters of the battery and PCM remain constant.
6: The parameters of the electrode are taken as the parameters of the battery in [1].

III. RESULTS

After the numerical is obtained, the process of the heat transfer is simulated by MATLAB. In this study, battery is considered up to grid number 5 and PCM from grid 6 to 10. The MATLAB programme is attached in the appendix. The data used in the simulation is in Table 1.

As seen in figure 3, without using PCM, the highest temperature in the battery is 55.7 °C after 3 hours. The temperature decreases from the center of the battery to the boundary.

According to Table 2, the simulated temperatures are quite close to the experimental temperatures reported by different researchers. Slight variation of results could be due to the use of different thermophysical properties in the simulation program.

Figure 4 presents the temperature distribution inside the battery and PCM. The temperature decreases along the radial direction. The temperature in the center of the battery is the highest, and the temperature of the boundary of PCM is the lowest. The highest temperature of the battery is 38.1°C. There is a drop of the highest temperature by 17.6 °C when PCM is used. For the heat generation rate of the battery used in the simulation programme, the temperature of PCM did not reach to the melting point resulting no melting of the PCM.
According to Table 3, the simulated temperature is slightly higher than the experimental temperature reported by Raminet al [5]. It may due to the use of different thermophysical properties in the simulation program. However, the simulation results are quite close to the experimental data available in the literature.

A sensitivity analysis was conducted to study the effect of different parameters on raise of battery temperature. The highest temperature of the battery in 5400s is compared.

After carefully analysis the data of Table 4 and 5 (in Appendix B), the following thermophysical properties of solid and liquid PCM were used for the following simulations: (a) The density of liquid PCM is assumed to be 100 kg/m^3less than the density of solid PCM (b) The thermal conductivity of liquid PCM is assumed to be half of the thermal conductivity of solid PCM (c) The heat capacity of liquid PCM is assumed to be 4/3 times of the heat capacity of solid PCM.
The graphs of the temperature with the parameters are in Appendix C.

For the battery with moderate heat generation rate, the trend of the maximum temperature raise of battery against the density of the battery is shown in Figure 5. The maximum temperature decreases as the density of the battery increases. Figure 6 shows the relationship between the maximum temperature raise of battery against the density of PCM. When the density of PCM increases, the maximum temperature of the battery decrease. The density of the battery and PCM has little effects on the maximum temperature raise of the battery. The maximum temperature against the thermal conductivity of the battery is presented in Figure 7. When the thermal conductivity increases, the maximum temperature raise of battery decreases first, and then increases. Figure 8 presents the effects of the thermal conductivity of PCM on the maximum temperature. The maximum temperature decreases when the thermal conductivity increases. The thermal conductivity of the battery and PCM are sensitive to the maximum temperature raise of the battery as the little increase of the thermal conductivity can change the temperature. Figure 9 and 10 shows the maximum temperature decreases as the specific heat of the battery or PCM increases, but the effect of the specific heat is little. Figure 11 shows that the latent heat of PCM has no effects on the temperature of the battery with moderate heat generation rate as there is no melting or very little melting for PCM. The effect of the melting temperature of PCM for the temperature is shown in Figure 12. The maximum temperature increases, then decreases as the melting temperature increases, and the melting points has little effects on the temperature of the normal battery. Figure 13 and 17 presents the effect of the convection coefficient on the temperature, and the temperature decreases as the increase of the convection coefficient. The convection coefficient has little effect as the convection coefficient is usually small. Figure 14 and 16 shows the increase of the radius and height will cause the sharp increase of the maximum temperature of the battery, and the radius and the height are sensitive parameters. The trend of the maximum temperature raise of the battery against the heat generation rate is presented in figure 15. The increase of heat generation rate will cause the increase of the maximum temperature, and it has a significant effect on the temperature.

As the thermal conductivity of PCM increases, the heat are easier to transfer out of PCM, and thus it helps to decrease the temperature of the battery significantly. When the thermal conductivity of the battery is very small, the increase of the thermal conductivity of the battery will make the heat easier to transfer out of the battery. However, further increase of the thermal conductivity of the battery while keep the thermal conductivity of PCM constant will block the heat transfer out of PCM, and thus the maximum temperature raise in the battery increase slightly. When the radius and height of the battery and PCM increases, the heat has longer distances to transfer, and thus the heat is more difficult to transfer out of the battery and PCM. When the heat generation rate per unit volume increases, there are more heat generated in the battery, and thus the temperature raise in battery will increase.

ρ_1 (kg/m^3)	c_1 (J/kgK)	k_1 (W/mK)	e(W/m^3)	$h_{up}/_{down}$ (W/m^2K)	l(m)	r_b(m)	ρ_2 (kg/m^3)	ρ_3 (kg/m^3)
2047	1360	0.4	11157	9	0.2	0.09	884	750
T_{amb}(°C)	T_{melt}(°C)	c_2(J/kgK)	c_3(J/kgK)	k_2(W/mK)	k_3(W/mK)	h(W/m^2K)	q(J/kg)	
25	31.5	2096	2761	2	1	5	153000	

Table 1: Parameters taken in the simulation

	Time (s)	Simulated results (°C)	Experimental results (°C)
Maximum temperature difference (°C)	1800	7.0	6.5[6] ,8.5[10]
	3000	11.2	13.5[6]
Minimum temperature difference (°C)	1800	1.5	4.5 [6] ,5.5[10]
	3000	3.2	9[6]
Average temperature difference (°C)	1800	6.6	5.5 [6] ,7[10]
	3000	10.1	11[6]

Table 2: Comparison of simulation results with experimental data on the temperature difference of the battery without PCM

	Time (s)	Simulated results (°C)	Experimental results (°C)
Maximum temperature difference(°C)	1800	6.3	6 [5]
	3600	9.6	7.5 [5]
	5400	11.3	8.5 [5]
	7200	12.2	10 [5]

Table 3: Comparison of simulation results with experimental data on the temperature difference of the battery with PCM

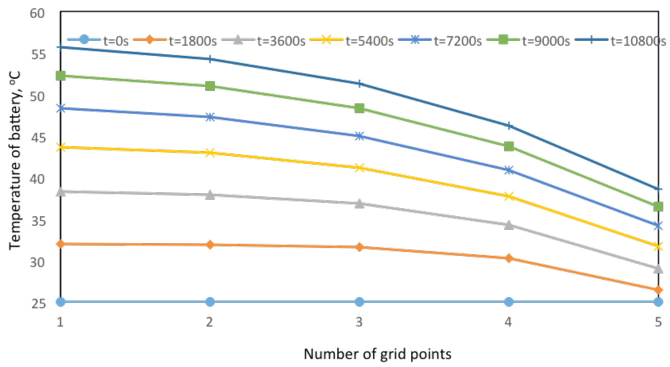

Figure 3: Distribution of temperature inside the battery with time when PCM is not used

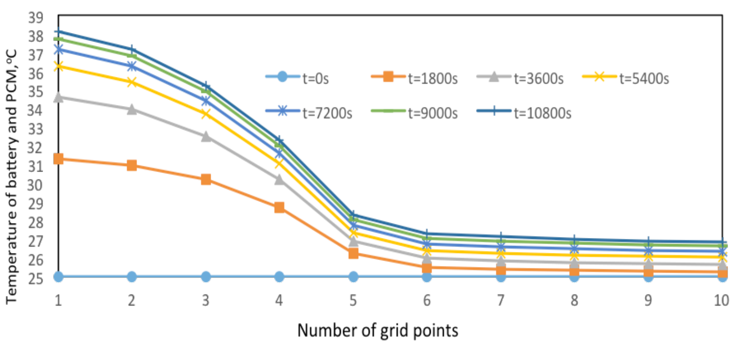

Figure 4: Distribution of temperature inside the battery and PCM with time when PCM is used

When the heat generation is relatively high, PCM will melt and absorb the heat generated by the battery. When e=300000 W/m^3, PCM will melt.

The temperature distribution inside the battery is shown in Figure 18. The temperature raise in the center of the battery is very high, and it is decreasing as the grid point increases. The first 3 grid of PCM finished melting, second last grid of PCM is melting, and the last grid of PCM did not start melting. The effects of the parameters are studied, and the highest temperature of the battery in 5400s is compared.

Figure 19 shows that the highest temperature decreases as the latent heat of PCM increases, and the latent heat is sensitive to the temperature when the heat generation rate is relatively high. The effect of the melting point of PCM on the temperature is shown in Figure 20. When the melting point of PCM increases, the highest temperature increases, then decreases, and the melting point of PCM is sensitive to the temperature. Figure 21 shows that the highest temperature decreases, then increases as the thermal conductivity of the battery increases, and the thermal conductivity is sensitive to the highest temperature of the battery.

The main effect of melting points on the maximum temperature due to the different amount of heat absorbed and transferred before and after PCM is melted. The parameters for solid and liquid PCM is different, and thus the maximum temperature will be different. When the latent heat of PCM is high, PCM can absorb more heat during melting and thus cool down the battery and thus the maximum temperature of the battery will decrease.

IV. DISCUSSION

The thermal conductivity is one of the most sensitive parameters for the temperature raise of the battery. In order to cool down the battery, PCM with high thermal conductivity is suitable. To increase the heat conductivity, hexagonal boron nitride (h-BN) nanosheets can be filled in paraffin-based composite phase change materials and the thermal conductivity of PCM can increase up to 60% [18]. Graphene and FLG fillers can also be added to paraffin-based PCM to increase the thermal conductivity strongly. The thermal conductivity can reach 10W/mK when 5% of the volume fraction of graphene-FLG filler is added [19]. Paraffin/xGnP composite PCMs [20] and nanoparticle-enhanced phase change materials [21] also have high thermal conductivity. β-Aluminium nitride powder can be added to the organic PCM to increase the thermal conductivity [22] and additives like graph are useful to increase the thermal conductivity of the shape-stabilized PCM [23].

V. CONCLUSION

Simulation results show that thermal conductivity of PCM and the battery, heat generation rate per unit volume, the radius of the battery and PCM, the height of the battery are sensitive parameters for the raise of battery temperature. For the battery with moderate heat generation rate, PCM will not melt, and PCM will act as a heat sink for cooling the Li-ion battery. The latent heat and the melting point of PCM have very little effect on the temperature of the battery. When the thermal conductivity of PCM is low, PCM may work as insulator to prevent the heat transfer, and the temperature of the battery may increase sharply. Thus, for cooling a normal battery, the material with high conductivity, high density and high heat capacity should be used. The battery with low and moderate heat generation rate, suitable conductivity, small radius and less height can be chosen in order to prevent the temperature from raising sharply.

When the heat generation rate is relatively high, PCM can be used to cool down the battery as PCM will melt and it will absorb a large amount of heat. The latent heat and the melting temperature of PCM are sensitive to the maximum temperature raise. The thermal conductivity, the radius and height of the battery and PCM become more sensitive to the maximum temperature raise compared to the normal battery, PCM with high thermal conductivity, high latent and suitable melting temperature should be selected for cooling the battery with high heat generation rate.

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Appendix A Nomenclature

k_1 Conductivity of the battery, W/mK
k_2 Conductivity of solid PCM, W/mK
k_3 Conductivity of liquid PCM, W/mK
ρ_1 Density of the battery, kg/m^3
ρ_2 Density of solid PCM, kg/m^3
ρ_3 Density of liquid PCM, kg/m^3
e Rate of heat generated by the battery per unit volume, W/m^3
Δr Radial distance of each control element, m
c_1 Specific heat of the battery, J/kgK
c_2 Specific heat of solid PCM, J/kgK
c_3 Specific heat of liquid PCM, J/kgK
q Latent heat of PCM, J/kg
h Convective heat transfer coefficient of the side surface of PCM, W/m^{2}K
h_{up} Convective heat transfer coefficient of the upper surface, W/m^{2}K
h_{down} Convective heat transfer coefficient of the bottom surface, W/m^{2}K
r_b Outer radius of the battery, m
r_{PCM} Outer radius of battery and PCM, m
l Height of the battery and PCM, m
E_{absorb} Energy absorbed by PCM at a particular instance, J
Q_i Total energy absorbed by the ith segment of PCM, J
T_i Temperature of ithsegment, ^{o}C
T_{melt} Melting temperature of PCM, ^{o}C
θ Angle of the section of the battery and PCM, rad
q_{in} Rate of heat transferred into the control element, J
q_{out} Rate of heat transferred out of the control element, J
q_{absorb} Rate of heat absorbed by the control element, J
E Rate of heat generation rate, J
k Thermal conductivity of an object, W/mK
A Area of an object, m^2
d Distance for the heat transfer, m
h_0 Convective coefficient, W/m^{2}K
ΔT Temperature difference, ^{o}C
$Q_{conduction}$ Rate of heat transfer by conduction
$Q_{convection}$ Rate of heat transfer by convection

Appendix B The parameters of battery and PCM taken from research paper

Reference	Density ρ_1 (kg/m^3)	Specific heat c_1(J/kgK)	Conductivity k_1(W/mK)	Heat generation rate over volume e (W/m^3)	Convective coefficient $h_{up}/_{down}$(W/m^2K)
[1]	2700	715	4		
[2]	2200	800	0.99		
[6]				11157	
[7]		1015			
[8]	2300	746			
[9]		1350			
[10]	2047	1360	0.4		9,45
[11]	2000	850			
[12]				9560.5	

Table 4: Properties of Li-ion battery used by different researchers

Reference	Type of PCM	Solid density ρ_2(kg/m^3)	Liquid density ρ_3 (kg/m^3)	Melting temperature T_{melt} (^oC)	Solid specific heat c_2 (J/kgK)	Liquid specific heat c_3(J/kg K)	Solid thermal conductivity k_2(W/mK)	Liquid thermal conductivity k_3(W/mK)	Convective coefficient h(W/m^2K)	Latent heat q(J/kg)
[4]	PCM/ Graphite composite	789			1980		16.6			185000
[5]	Capric acid	884		31.5	2096	2761	2			153000
[5]	Eicosane	778		36.8	2132	2350	0.27			241000
[5]	$Na_2SO_4.10H_2O$	1485		32.4	2093	3349	0.544			254000
[11]									5(natural), 15(forced)	
[13]	Paraffin	870	750	28	1800	2400	0.2			179000
[13]	Paraffin	1380		26	2500		0.6			180000
[14]	paraffin	789	750		1800	2400	0.18	0.19		175066
[15]	Parafin wax	916	790	64			0.346	0.167		173600

[15]	Erythritol	1480	1300	118			0.733	0.326		339800
[15]	Naphthalene	1145	976	80			0.341	0.132		147700
[15]	Caprylic acid	981	901	16			0.149			148500
[16]	Paraffin	850	775		900		0.2			214400
[17]	$Ba(OH)_2 \cdot 8H_2O$	2070	1937	48			1.225	0.653		265700
[17]	$Mg(NO_3)_2 \cdot 6H_2O$	1636	1550	89			0.611	0.490		162800
[17]	$MgCl_2 \cdot 6H_2O$	1569	1450	117			0.694	0.570		168600
[17]	Palmitic acid	989	850	64				0.162		185400
[17]	Polyglycol E600	1232	1126	22				0.189		127200

Table 5: Properties of PCM used by different researchers

Appendix C The Graph of the highest temperture with the different parameters

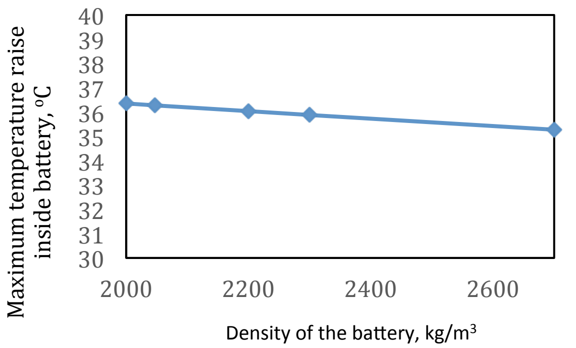

Figure 5: Variation of maximum temperature raise inside battery with density of the battery

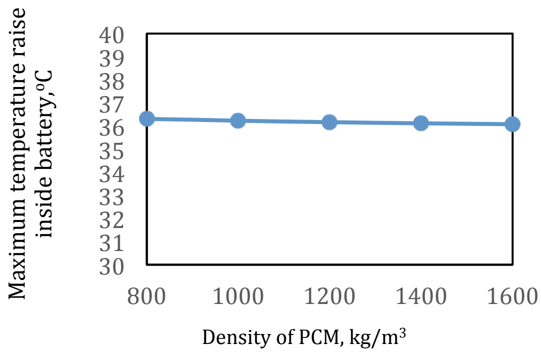

Figure 6: Variation of maximum temperature raise inside battery with density of PCM

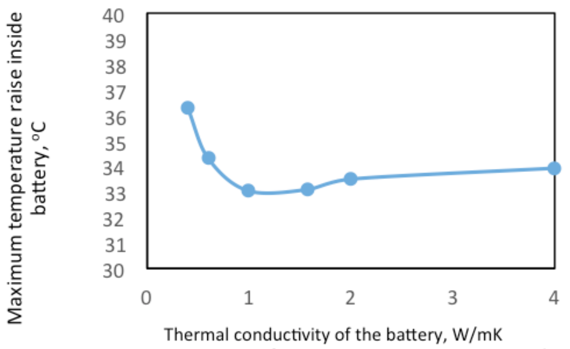

Figure 7: Variation of maximum temperature raise inside the battery with the thermal conductivity of the battery

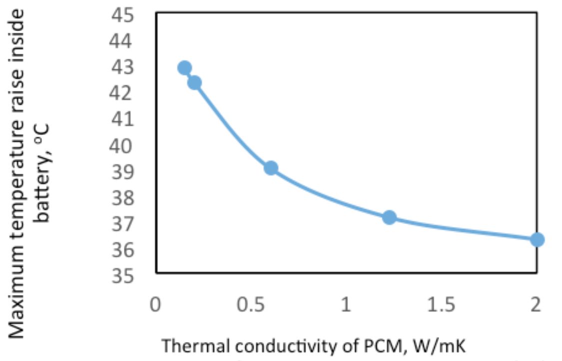

Figure 8: Variation of maximum temperture raise inside the battery with the thermal conductivity of PCM

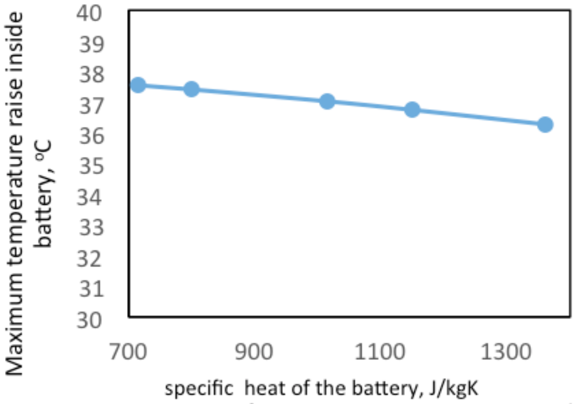

Figure 9: Variation of maximum temperature raise inside battery with specific heat of the battery

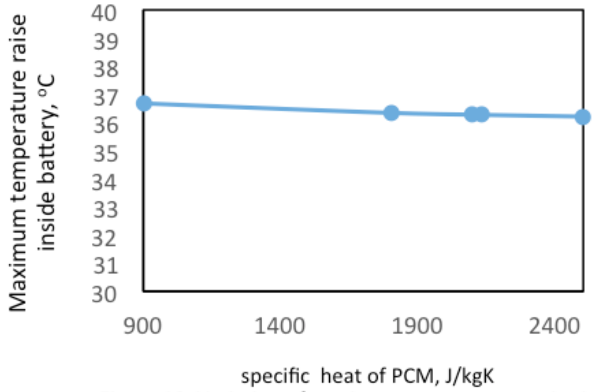

Figure 10: Variation of maximum temperature raise inside battery with specific heat of PCM

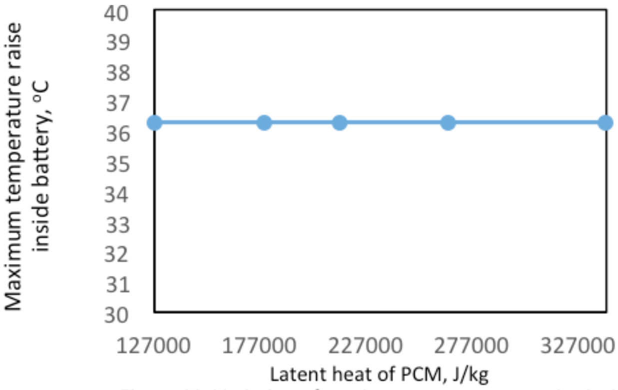

Figure 11: Variation of maximum temperature raise inside battery with latent heat of PCM

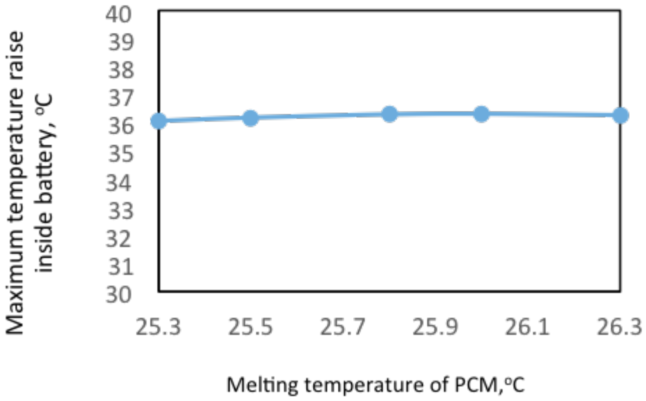

Figure 12: The trend of highest temperature against the melting temperature of PCM

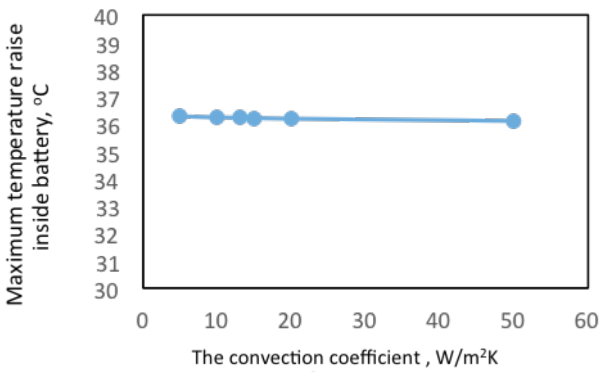

Figure 13: Variation of maximum temperature raise inside battery with convective heat transfer coefficients

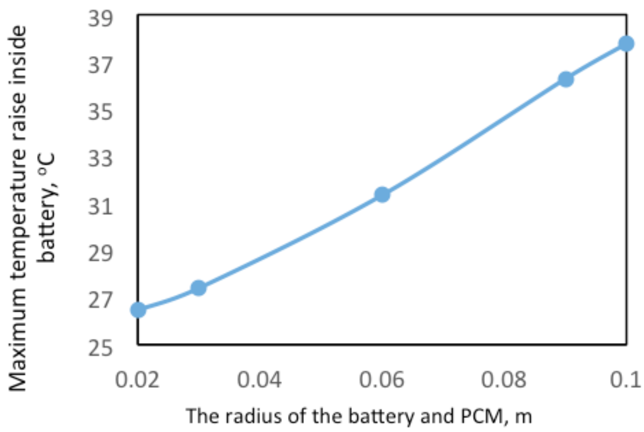

Figure 14: Variation of maximum temperature raise inside battery with the radius of the battery

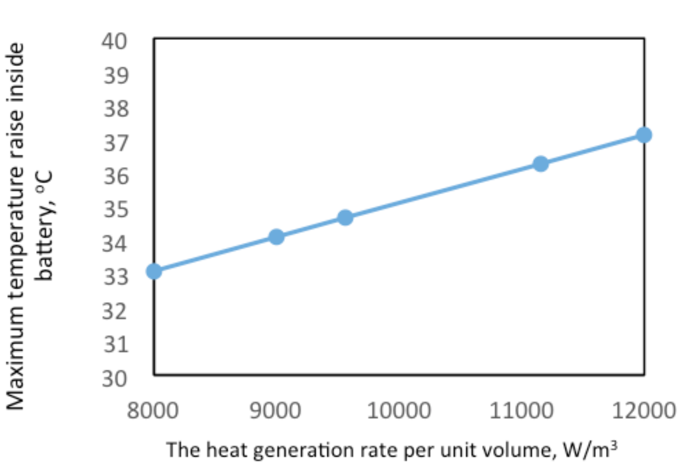

Figure 15: Variation of maximum temperature raise inside battery with the heat generation rate per unit volume

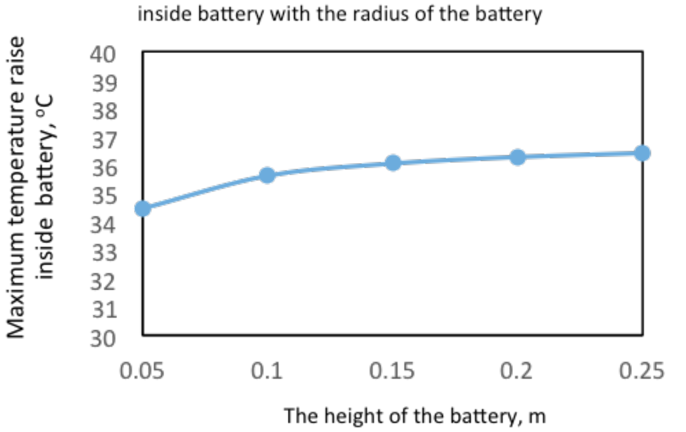

Figure 16: Variation of maximum temperature raise inside battery with the height of the battery

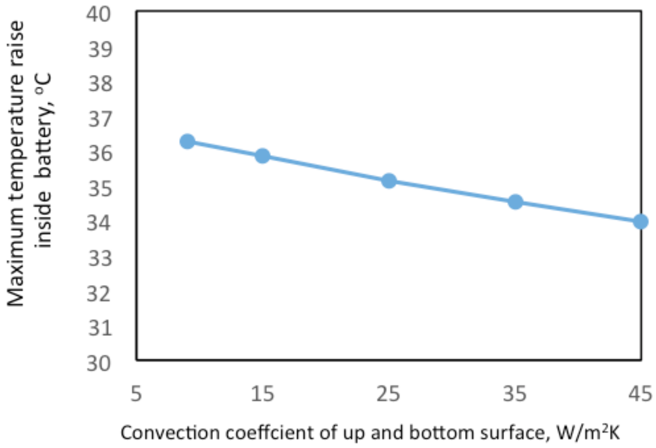

Figure 17: Variation of maximum temperature raise inside battery with convective heat transfer coefficients

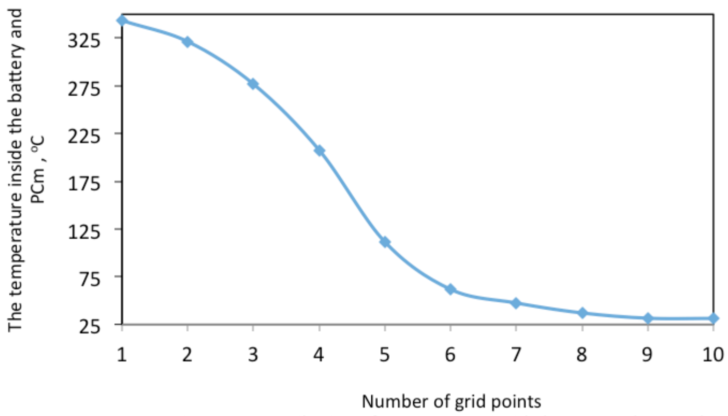

Figure 18: Distribution of temperature inside battery and PCM when $e=300000W/m^3$ in 5400s

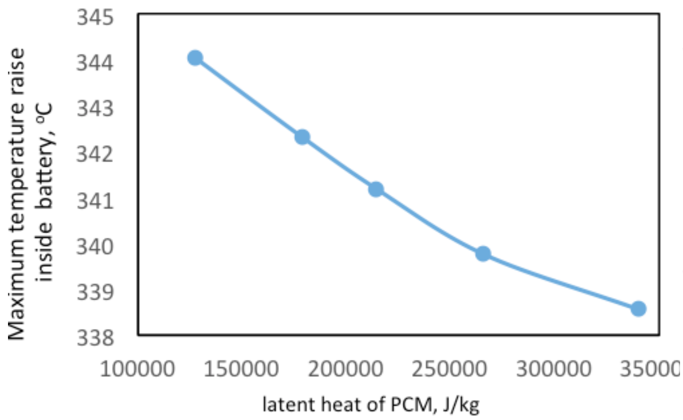

Figure 19: Variation of maximum temperature raise inside battery with latent heat of PCM when $e=300000W/m^3$

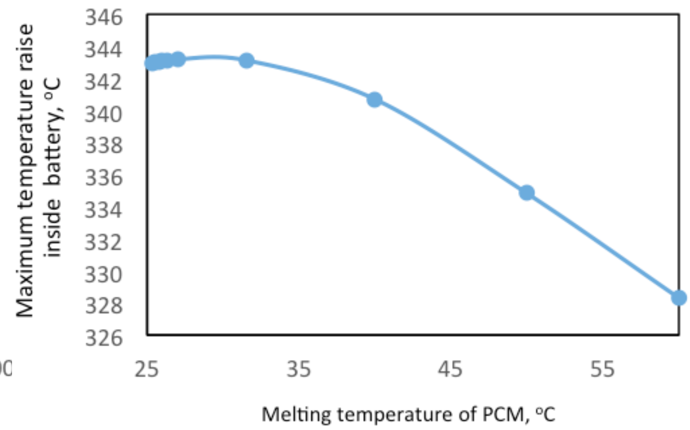

Figure 20: Variation of maximum temperature raise inside battery with melting temperature of PCM when $e=300000W/m^3$

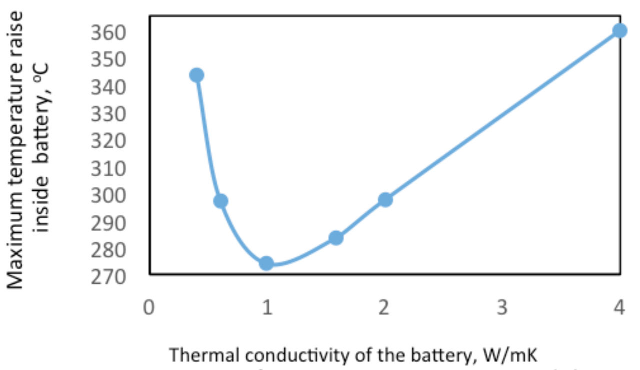

Figure 21: Variation of maximum temperature raise inside battery with thermal conductivity of battery when $e=300000W/m$

Appendix D The MATLAB programme

```
clear all
format long
k1=0.4 % thermal conductivity of the battery, W/(m oC)
k2=2  % thermal conductivity of solid PCM, W/(m oC)
k3=1 % thermal conductivity of liquid PCM, W/(m oC)
l1=0.09 % radius of the battery and PCM,m
l2=0.2 % height of the battery and PCM,m
h=5  % convective coefficient,W/(m^2 oC)
hup=9 % convective coefficient of the upper surface,W/(m^2 oC)
hdown=9 % convective coefficient of the bottom surface,W/(m^2
oC)
ro1=2047 % density of the battery,kg/m^3
ro2=884  % density of solid PCM,kg/m^3
ro3=750  % density of liquid PCM,kg'/m^3
Tamb=25    % The ambient temperature,oC
c1=1360     % specific heat of the battery,J/(kg oC)
c2=2096    % specific heat of solid PCM,J/(kg oC)
c3=2761    % specific heat of liquid PCM,J/(kg oC)
n=5 % number of segments of the battery
p=10 % number of total segments
dr=l1/p % length of each segment,m
Tmelt=31.5 % melting point of PCM,oC
q=153000 % latent heat of PCM, J/kg
dx=l2 % height of each segment ,m
e=11157 %  heat generation rate per unit volume, W/m^3
km=(k2+k3)/2   % conductivity when PCM is melting
for i=1:p % make the matrix T (initial temperature)
   T(i)=Tamb
end
  T=T'
for i=1:n
   S(i)=(e*dx+hup*Tamb+hdown*Tamb)/ro1/c1/dx
end
for i=n+1:p-1 % make the matrix S
       S(i)=(hup*Tamb+hdown*Tamb)/ro2/c2/dx
end
   S(p)=2*p*h*Tamb/(2*p-
1)/ro2/dr/c2+(hup*Tamb+hdown*Tamb)/ro2/c2/dx
S=S'
for i=n+1:p
        Q(i)=0
        ea(i)=0
end
m=0
t=0
y=1
axis([1 p 25 45])
j=[1,2:length(T)]
T_plot=[T']
plot(j,T_plot)
```

```
text(j(2),T_plot(2),['t=',int2str(t),'s'])
xlabel('point number,i')
ylabel('T^oC')
hold on
for r=1:3
for r1=1:1800
% before melting
if T(n+1)<Tmelt
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k2/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k2/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c2/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c2/dr/dr-
(2*n+2)*k2/(2*n+1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
    A(n+1,n+2)=(2*n+2)*k2/(2*n+1)/ro2/c2/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
for i=n+2:p-1
        A(i,i-1)=2*k2*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*k2/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
end
end
% The n+1 segment is melting
if T(n+1)>=Tmelt
if Q(n+1)<q*ro2*(2*n+1)*dr*dr/2 % The condition of melting
 ea(n+1)=k1*n*(T(n)-T(n-1))-k2*(n+1)*(T(n+1)-T(n))-
(hup+hdown)*(2*n+1)*dr*dr*(T(n+1)-Tamb)/2/dx % The heat
absorbed by the segment,J
     Q(n+1)=Q(n+1)+ea(n+1) % The total heat absorbed,J
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
```

```
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*km/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*km/(2*n-1)/ro1/c1/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
    i=n+2
     A(i,i-1)=2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr-2*k2*i/(2*i-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
for i=n+3:p-1
        A(i,i-1)=2*k2*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*k2/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
end
    S(n+1)=0
end
end
% segment (n+1) finish melting
if Q(n+1)>=q*ro2*(2*n+1)*dr*dr/2 % condition of finish melting
of (n+1) segment
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k2/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k2/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
```

```
    i=n+2
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr+2*k2*i/(2*i-
1)/ro2/c2/dr/dr)-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
for i=n+3:p-1
        A(i,i-1)=2*k2*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*k2/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
end
    S(n+1)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
% segment n+2 is melting
if T(n+2)>=Tmelt
if Q(n+2)<q*ro2*(2*n+3)*dr*dr/2 % The condition of melting
    ea(n+2)=k3*(n+1)*(T(n+1)-T(n+2))-k2*(n+2)*(T(n+2)-T(n+3))-
(hup+hdown)*(2*n+3)*dr*dr*(T(n+2)-Tamb)/2/dx  % The heat
absorbed by the segment,J
    Q(n+2)=Q(n+2)+ea(n+2) % The total heat absorbed,J
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*km/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*km/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
    i=n+3
     A(i,i-1)=2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr-2*k2*i/(2*i-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
for i=n+4:p-1
        A(i,i-1)=2*k2*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*k2/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
```

```
end
    S(n+1)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
    S(n+2)=0
end
end
% segment n+2 finish melting
if Q(n+2)>=q*ro2*(2*n+3)*dr*dr/2 % condition of finish melting
of n+2 segment
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
    i=n+2
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr+2*k2*i/(2*i-
1)/ro2/c3/dr/dr)-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c3/dr/dr
          i=n+3
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr+2*k2*i/(2*i-
1)/ro2/c2/dr/dr)-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
for i=n+4:p-1
        A(i,i-1)=2*k2*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*k2/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
end
   S(n+1)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
   S(n+2)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
% The n+3 segment is melting
if T(n+3)>=Tmelt
```

```
if Q(n+3)<q*ro2*(2*n+5)*dr*dr/2 % The condition of melting
    ea(n+3)=k3*(n+2)*(T(n+2)-T(n+3))-k2*(n+3)*(T(n+3)-T(n+4))-
(hup+hdown)*(2*n+5)*dr*dr*(T(n+3)-Tamb)/2/dx  % The heat
absorbed by the segment,J
    Q(n+3)=Q(n+3)+ea(n+3) % The total heat absorbed,J
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
     A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
    i=n+2
    A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr-2*km*i/(2*i-
1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*km*i/(2*i-1)/ro2/c3/dr/dr
for i=n+4
        A(i,i-1)=2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-2*km*(i-1)/(2*i-1)/ro2/c2/dr/dr-2*k2*i/(2*i-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
end
end
            S(n+1)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
   S(n+2)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
   S(n+3)=0
end
% segment n+3 finish melting
if Q(n+3)>=q*ro2*(2*n+5)*dr*dr/2 % condition of finish melting
of n+2 segment
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
```

```
end
     A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k2/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
     i=n+2
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr+2*k3*i/(2*i-
1)/ro2/c3/dr/dr)-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k3*i/(2*i-1)/ro2/c3/dr/dr
        i=n+3
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr+2*k2*i/(2*i-
1)/ro2/c3/dr/dr)-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c3/dr/dr
    i=n+4
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr
        A(i,i)=-(2*k3*(i-1)/(2*i-1)/ro2/c2/dr/dr+2*k2*i/(2*i-
1)/ro2/c2/dr/dr)-(hup+hdown)/ro2/c2/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c2/dr/dr
for i=n+1:n+3
        S(i)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
end

% segment p-1 is melting
if T(p-1)>=Tmelt
if Q(p-1)<q*ro2*(2*p-3)*dr*dr/2-(hup+hdown)*(2*p-
3)*dr*dr*(T(p-1)-Tamb)/2/dx % The condition of melting
    ea(p-1)=k3*(p-2)*(T(p-2)-T(p-1))-k2*(p-1)*(T(p-1)-T(p))   %
The heat absorbed by the segment,J
    Q(p-1)=Q(p-1)+ea(p-1)  % The total heat absorbed,J
for i=1:p   % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
```

```
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*km/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*km/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
for i=n+2:p-3
        A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k3*i/(2*i-1)/ro2/c3/dr/dr
end
    i=p-2
     A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr-2*km*i/(2*i-
1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*km*i/(2*i-1)/ro2/c3/dr/dr
for i=n+1:p-2
           S(i)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
   S(p-1)=0
end
end
% segment p-1 finish melting
if Q(p-1)>=q*ro2*(2*p-3)*dr*dr% condition of finish melting of
w segment
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
```

```
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k3/(2*p-1)/ro2/c2/dr/dr
    A(p,p)=-2*(p-1)*k3/(2*p-1)/ro2/c2/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c2/dr/dr-(hup+hdown)/ro2/c2/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
for i=n+2:p-2
        A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k3*i/(2*i-1)/ro2/c3/dr/dr
end
    i=p-1
    A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr-2*k2*i/(2*i-
1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k2*i/(2*i-1)/ro2/c3/dr/dr
for i=n+1:p-1
            S(i)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
end
% The segment p is melting
if T(p)>=Tmelt
if Q(p)<q*ro2*(2*p-1)*dr*dr/2 % The condition of melting
    ea(p)=k3*(p-1)*(T(p-1)-T(p))-h*p*dr*(T(p)-Tamb)-
(hup+hdown)*(2*p-1)*dr*dr*(T(p)-Tamb)/2/dx % The heat absorbed
by the segment,J
    Q(p)=Q(p)+ea(p) % The total heat absorbed,J
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
     A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
for i=n+2:p-2
```

```
        A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k3*i/(2*i-1)/ro2/c3/dr/dr
end
     i=p-1
    A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr-2*km*i/(2*i-
1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*km*i/(2*i-1)/ro2/c3/dr/dr
for i=n+1:p-1
        S(i)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
        S(p)=0
end
end
% after melting
if Q(p)>=q*ro2*(2*p-1)*dr*dr% The condition after melting
for i=1:p  % Make the matrix A
for j=1:p
        A(i,j)=0
end
end
    A(1,1)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(1,2)=2*k1/ro1/c1/dr/dr
    A(n,n-1)=2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr
    A(n,n)=-2*(n-1)*k1/(2*n-1)/ro1/c1/dr/dr-2*n*k3/(2*n-
1)/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
    A(n,n+1)=2*n*k3/(2*n-1)/ro1/c1/dr/dr
    A(n+1,n)=2*n*k1/(2*n+1)/ro2/c3/dr/dr
    A(n+1,n+1)=-2*n*k1/(2*n+1)/ro2/c3/dr/dr-
(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
    A(n+1,n+2)=(2*n+2)*k3/(2*n+1)/ro2/c3/dr/dr
    A(p,p-1)=2*(p-1)*k3/(2*p-1)/ro2/c3/dr/dr
    A(p,p)=-2*(p-1)*k3/(2*p-1)/ro2/c3/dr/dr-2*p*h*dr/(2*p-
1)/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
for i=2:n-1
        A(i,i-1)=2*k1*(i-1)/(2*i-1)/ro1/c1/dr/dr
        A(i,i)=-2*k1/ro1/c1/dr/dr-(hup+hdown)/ro1/c1/dx
        A(i,i+1)=2*k1*i/(2*i-1)/ro1/c1/dr/dr
end
for i=n+2:p-1
        A(i,i-1)=2*k3*(i-1)/(2*i-1)/ro2/c3/dr/dr
        A(i,i)=-2*k3/ro2/c3/dr/dr-(hup+hdown)/ro2/c3/dx
        A(i,i+1)=2*k3*i/(2*i-1)/ro2/c3/dr/dr
end
for i=n+1:p-1
        S(i)=(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
        S(p)=2*p*h*Tamb/(2*p-
1)/ro2/dr/c3+(hup*Tamb+hdown*Tamb)/ro2/c3/dx
end
```

```
%plot the graph
m=m+1
K1=y*(A*T+S)
K2=y*(A*(T+K1/2)+S)
K3=y*(A*(T+K2/2)+S)
K4=y*(A*(T+K3)+S)
T=T+(K1+2*K2+2*K3+K4)/6
t=y*m
end
j=[1,2:length(T)]
T_plot=[T']
axis([1 p 25 45])
T_plot=[T']
plot(j,T_plot)
text(j(2),T_plot(2),['t=',int2str(t),'s'])
hold on
end
```